[15]Jain, A.C. and Adimurthy, V., "Hypersonic Merged Stagnation Shock Layers, Part II, Cold Wall Case," *AIAA Journal*, Vol. 12, March 1974, pp. 348-354.

[16]Ahouse, D.R. and Bogdonoff, S.M., "An Experimental Flow Field Study of the Rarefied Blunt Body Problem," AIAA Paper 69-656, 1969, San Francisco, Calif.

[17]Li, C.P., "Hypersonic Nonequilibrium Flow Past a Sphere at Low Reynolds Numbers," AIAA Paper 74-173, Feb. 1974, Washington, D.C.

[18]Wittliff, C.E. and Wilson, M.R., "Low-Density Stagnation-Point Heat Transfer in the Hypersonic Shock Tunnels," *ARS Journal*, Vol. 32, 1962, pp. 275-276.

[19]Potter, L.J. and Miller, J.T., "Total Heating Load on Blunt Axisymmetric Bodies in Low Density Flow," *AIAA Journal*, Vol. 1, 1963, pp. 480-481.

Structural Optimization under Combined Blast and Acoustic Loading

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Introduction

CONSIDERABLE amount of work has been reported in the literature on the minimum weight design of structures subjected to static and dynamic response restrictions.[1] The optimization of structures in a random vibration environment was discussed in Ref. 2. The minimum cost design of structures with random material properties has also been considered.[3] The present Note deals with the optimum design of beam-type and plate-type structures subjected to a constraint on the probability of failure due to combined blast and finite-duration acoustic loading. The present problem has application in the design of structures located near rocket testing or launching facilities where the structures may occasionally be exposed to blast loads due to explosions in addition to experiencing rocket-noise excitation. The expressions for the first passage probabilities of a linear oscillator are used in determining the damage probability of the given structure. The optimization problem is stated as a constrained nonlinear programing problem. The design of a simply supported rectangular plate is considered as an example problem.

Optimization Problem

The problem of optimum design of a structure subjected to combined blast and acoustic loading can be stated as a standard nonlinear programing problem as

Find $\boldsymbol{D}$ such that $f(\boldsymbol{D})\rightarrow$ minimum and $g_j(\boldsymbol{D})\leq 0, j=1,2,\ldots,m$

where $\boldsymbol{D}$ is the vector of design variables, $f(\boldsymbol{D})$ is the objective function and $g_j(\boldsymbol{D})$ are the constraint functions. The objective of the present work is to minimize the weight, and the major behavior constraint is that the probability of failure under the stated loading condition must be less than or equal to a specified value. The other constraints considered in this work include upper and lower limits on the fundamental natural frequency of the structure, upper and lower bounds on the design variables, and an upper bound on the time interval (T_c) required for the response autocorrelation function of the structure to decay to a small value. While the constraints on the fundamental frequency and the design variables are common for any dynamic structural optimization problem, the constraint on T_c becomes necessary in the present problem in view of the relations used for the failure probability of the structure.

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Probability of Failure

It is well established that the stresses and deformations associated with the fundamental flexural modes of beam-like and plate-like structures exposed to acoustic pressures generally predominate over those associated with the higher modes. Thus an engineering estimate of the probability of failure can be obtained by idealizing the structure as a single degree-of-freedom oscillator corresponding to its fundamental mode.

Probability of Failure due to Individual Loads

The pressure vs time curves corresponding to blast loads generally rise very sharply and decay slowly. The displacement response $z(t)$ of a single degree-of-freedom oscillator to an exponentially decaying blast load $F_0\cdot e^{-\beta t}$ is given by

$$\frac{z(t)}{z_s}\left[\left(\frac{\beta}{\omega}\right)^2-2\zeta\frac{\beta}{\omega}+1\right] = \left[\frac{\beta-\zeta\omega}{\omega(1-\zeta^2)^{1/2}}\right]e^{-\zeta\omega t}\cdot\sin\omega\sqrt{(1-\zeta^2)}\,t - e^{-\zeta\omega t}\cdot\cos\omega(1-\zeta^2)^{1/2}t+e^{-\beta t} \quad (1)$$

where $\omega=(K/M)^{1/2}=$ circular natural frequency of the oscillator, K=stiffness, M=mass, $z_s=(F_0/K)$ =static deflection under a force F_0, β= decay exponent, and t=time. Equation (1) assumes that the oscillator is at rest and equilibrium at $t=0$. Thus, if the oscillator (structure) is excited by a blast load only, failure will occur if the maximum value of $z(t)$ given by Eq. (1) exceeds the prescribed limit Z in the time interval T.

If the structure is exposed to acoustic excitation only, the probability of failure in a time interval T can be calculated by considering the problem as a first passage problem.[4] In the present work, the random acoustic excitation and hence the response is assumed to be a Gaussian random process. Then the probability P that the absolute value of the displacement of a linear oscillator will exceed a threshold value Z in a time interval T is approximately given by[4]

$$P[\,|z(t)|>Z]=P(Z,T)=1-Ae^{-\alpha fT} \quad (2)$$

where f is the natural frequency of the oscillator, and A and α are functions of the threshold level Z, the conditions of motion at the beginning of the time interval T, and the damping of the oscillator. The values of A and α can be obtained for any specified Z.[4] It is to be noted that Eq. (2) holds true for time intervals $T>T_c$, where T_c denotes the time interval required for the response autocorrelation function to decay to a small value. The expression for T_c may be taken as

$$T_c\simeq 1/(2\pi f\zeta) \quad (3)$$

where ζ is the damping ratio of the oscillator.

Probability of Failure When Both Loads Act Simultaneously

It can be seen that for a slowly decaying force ($\beta/\omega<<1$) and for an oscillator with negligible damping, Eq. (1) may be approximated as

$$[z(t)/z_s]\,[1+(\beta/\omega)^2]=1-\cos\omega t \quad (4)$$

On the other hand, for a rapidly decaying force ($\beta/\omega>>1$), Eq. (1) becomes

$$[z(t)/z_s]\,\,[1+(\beta/\omega)^2]=(\beta/\omega)\sin\omega t \quad (5)$$

Thus for very large and for very small values of β/ω, the right hand sides of Eqs. (4) and (5) can be seen to be sinusoids. To obtain an engineering estimate of the probability of damage, the right hand sides of Eqs. (4) and (5) can be approximated by square waves. This means that the response of the structure for a slowly decaying blast load will be at its maximum value z_{max} for half of the time and at its minimum value z_{min} for half of the time. When both the blast and acoustic loads act simultaneously, failure of the structure may occur when the response due to the pulse load is either at z_{max} or at z_{min} where

$$z_{max} = \max[z_s(1-\cos\omega t)/1+(\beta/\omega)^2] \simeq 2z_s = (2F_0/K) \quad (6)$$

and

$$z_{min} = \min[z_s(1-\cos\omega t)/1+(\beta/\omega)^2] \simeq 0 \quad (7)$$

When the blast response $z_b(t)$ is at z_{max}, the probability that the total response due to the combined loading, $z_t(t)$, will exceed the threshold value Z is same as the probability that the positive portion of the acoustic response, $z_a(t)$, will exceed the value $(Z-z_{max})$. Since for a random process having a mean value of zero, $+z(t)$ is as likely to exceed $+Z$ as $-z(t)$ is to exceed $-Z$, the probability of failure is given by

$$P_f|z_b(t) = z_{max} = P[z_t(t) > Z] = P[z_a(t) > Z - z_{max}] + P[-z_a(t) < -Z - z_{max}] = \tfrac{1}{2}P(Z-z_{max}, T) + \tfrac{1}{2}P(Z+z_{max}, T) \quad (8)$$

When the blast response is at z_{min}, the probability of failure in a time interval T is given by

$$P_f\Big|_{zb(t)=z_{min}} = P[z_a(t) > Z] = P(Z,T) \quad (9)$$

Thus the total failure probability can be expressed as

$$P_{f_{combined}} = P_f\Big|_{zb(t)=z_{max}} \cdot P[z_b(t) = z_{max}] + P_f\Big|_{zb(t)=z_{min}} \cdot P(z_b(t) = z_{min}] \quad (10)$$

Since the probability of occurrence of each of the events $z_b(t) = z_{max}$ and $z_b(t) = z_{min}$ is ½, the probability of failure of the structure under the combined loading is given by

$$P_{f_{combined}} = \tfrac{1}{4}P(Z-z_{max}, T) + \tfrac{1}{4}P(Z+z_{max}, T) + \tfrac{1}{2}P(Z,T) \quad (11)$$

In the case of a rapidly decaying force pulse $(\beta/\omega >> 1)$, the maximum and minimum values of the response are given by

$$z_{max} = -z_{min} = \max[z_s(\beta/\omega)\sin\omega t/1+(\beta/\omega)^2] \simeq (F_0/K)\cdot(\omega/\beta) \quad (12)$$

By proceeding as in the case of a slowly decaying force, the probability of failure in this case due to the combined loading can be obtained as

$$P_{f_{combined}} = \tfrac{1}{2}P(Z-z_{max}, T) + \tfrac{1}{2}P(Z+z_{max}, T) \quad (13)$$

Probability of Failure Due to Blast after Acoustic Loading

In the case of a rocket launch or test, the structure near the launch site will be exposed initially to an acoustic excitation (rocket noise) for a time period T_1. If the launch or test is terminated by an explosion, the structure will be subjected to a blast load in addition to the continuation of the rocket noise for a time period T_2. Then the probability of failure due to the acoustic loading is given by

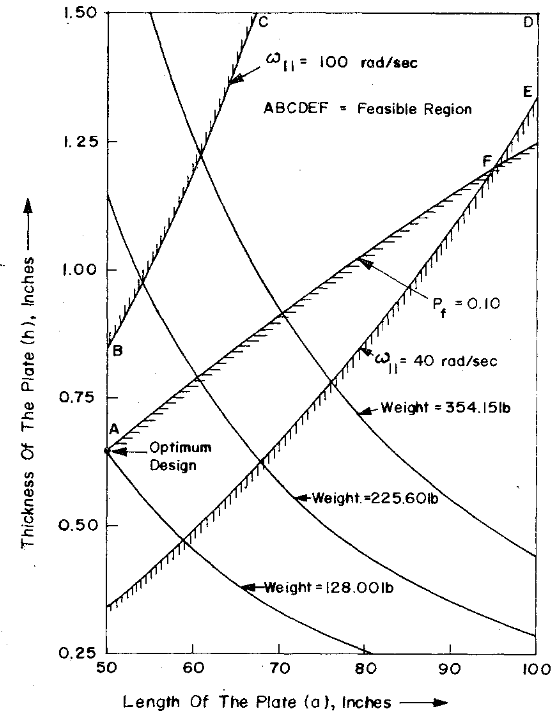

Fig. 1 Optimum design of the plate by graphical method.

$$P_{f_{acoustic}} = P(Z, T_1) \quad (14)$$

and the probability of failure due to the combined loading is given by Eq. (11) or (13). Since Eqs. (11) and (13) are applicable only if $P_{f_{acoustic}} < 1$, the overall probability of failure of the structure is given by

$$P_f = P(Z,T_1) + \tfrac{1}{4}[1-P(Z,T_1)][P(Z-z_{max},T_2) + P(Z+z_{max},T_2) + 2P(Z,T_2)] \text{ for } (\beta/\omega) << 1 \quad (15)$$

and

$$P_f = P(Z,T_1) + \tfrac{1}{2}[1-P(Z,T_1)][P(Z-z_{max},T_2) + P(Z+z_{max}, T_2)] \text{ for } (\beta/\omega) >> 1 \quad (16)$$

Numerical Example

The minimum weight design of a simply supported rectangular laminated plate of aspect ratio $a/b = 1.25$ is considered as an example problem. The plate is assumed to be made up of five equally thick laminations. The material of the plate is assumed as glass with a density of 170 lb/ft^3 and a maximum allowable stress of 6000 psi. The octave-band sound pressure levels expected at the site of the structure are taken as shown in Table 1. The time period T_1 during which only the acoustic excitation persists is 20 sec, and the time period T_2 during which both the acoustic and the blast loads act is 2.5 sec. The blast load is given by $0.2\,e^{-2t}$ psi.

Table 1 Octave-band sound pressure levels[8]

Octave band center frequency (Hz)	4	8	16	31.5	63
SPL (db re $2\times10^{-4}\,\mu$bar)	136	136	135	134	121

The design variables are the length (a'') and the thickness (h'') of the plate. The objective function to be minimized is given by $f(a,h) = 0.0787\ a^2h$ lbs. The damping ratio corresponding to the fundamental mode is assumed as $\zeta = 0.01$, and the upper and lower bounds on the fundamental frequency are taken as 40.0 and 100.0 rad/sec, respectively. In addition, the design variables are restricted as $50'' \leq a \leq 100''$ and $0.25'' \leq h \leq 1.50''$.

In this case, the fundamental mode shape is given by[5]

$$\phi_{11}(x,y) = \sin\frac{\pi x}{a}\cdot\sin\frac{\pi y}{b} \tag{17}$$

and the corresponding frequency by

$$\omega_{11} = 2\pi f = \pi^2\sqrt{Eh^3/12\mu(1-\nu^2)}\left[\frac{1}{a^2}+\frac{1}{b^2}\right] \tag{18}$$

For a uniform load $F(t)$ acting on the plate, the modal force, $G(t)$, is given by

$$G(t) = 4ab/\pi^2\cdot F(t) \tag{19}$$

The modal mass and modal stiffness are given by

$$M = \mu S \sin\pi x/a\cdot\sin\pi y/b \tag{20}$$

and

$$K = \omega_{11}^2\cdot M \tag{21}$$

The maximum stress (s) induced under the maximum permissible central deflection Z is given by[6]

$$s = \frac{\pi^2 EhZ}{2(1-\nu^2)}\left[\frac{\nu}{a^2}+\frac{1}{b^2}\right] \tag{22}$$

Here E denotes the Young's modulus, μ the mass per unit area, ν the Poisson's ratio, and S the surface area of the plate.

Once the plate is modelled as a single degree of freedom oscillator, the root mean square displacement of the oscillator, $z_{\rm rms}$, under the excitation of a Gaussian noise can be expressed as[7]

$$z_{\rm rms} = (1/2K)\sqrt{\pi W_F\cdot f/\zeta} \tag{23}$$

where W_F is the spectral density of the exciting forces given by

$$W_F = G^2{\rm rms}/\Delta f \tag{24}$$

with $G_{\rm rms}$ representing the root mean squarse value of the modal exciting force and Δf denoting the band width of the forcing frequency. In the case of octave band sound pressure levels with central frequency f_c, Δf is given by

$$\Delta f = f_c/\sqrt{2} \tag{25}$$

Equations (17) to (25) can be used to evaluate the probability of failure and other constraints. The feasible region, in which all the constraints are satisfied, is shown in the two-dimensional design space of Fig. 1. The contours of the objective function are also shown in the figure and the minimum weight can be seen to be 128.0 lb. The optimum values of the design variables are $h_{\rm opt} = 0.65''$ and $a_{\rm opt} = 50.0''$.

Conclusion

The problem of optimization of beam-like and plate-like structures with a constraint on the probability of failure due to combined acoustic and blast loading has been stated in the form of a standard nonlinear programing problem. The probability of failure has been estimated by approximating the response of the structure by that of a simple oscillator corresponding to the fundamental mode of the structure. The accuracy of the present analysis can be improved by approximating the structural behavior by several mode shapes. The number of design variables can be taken more than two and standard mathematical programing techniques can be applied to solve the problem.

References

[1] Rao, S.S., "Optimization of Complex Structures to Satisfy Static, Dynamic and Aeroelastic Requirements, *International Journal of Numerical Methods in Engineering,* Vol. 8, No. 2, 1974, pp. 249-269.

[2] Nigam, N.C., "Structural Optimization in Random Vibratin Environment," *AIAA Journal,* Vol. 10, April 1972, pp. 551-553.

[3] Rao, S.S., "Minimum Cost Design of Concrete Beams with a Reliability-Based Constraint," *Building Science,* Vol. 8, No. 1, 1973, pp. 33-38.

[4] Crandall, S.H., Chandiramani, K.L., and Cook, R.G., "Some First Passage Problems in Random Vibrations," *Journal of Applied Mechanics, Transactions of ASME,* Vol. 33, No. 3, 1966, pp. 532-538.

[5] Rothbart, H.A., (ed.), *Mechanical Design and Systems Handbook,* McGraw-Hill, N.Y., 1964.

[6] Timoshenko, S.P. and Woinowsky-Krieger, S., *Theory of Plates and Shells,* McGraw-Hill, N.Y., 1959.

[7] Crandall, S.H., and Mark, W.D., *Random Vibrations in Mechanical Systems,* Academic Press, N.Y., 1963.

[8] Ungar, E.E., and Kadman, Y., "Estimation of Structural Damage from Combined Blast and Finite Duration Acoustic Loading," *Shock and Vibration Bulletin,* Vol.39, Part 3, 1969, pp.65-70.

Liquid Drop Acceleration and Deformation

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Introduction

It is well known[1-11] that when a liquid drop is subjected to rapid accelation by an air stream, lateral deformation and breakup are subsequent. The problems of aerodynamic stripping and catastrophic breakup are so overwhelming, that the simple dynamics of the liquid drop rely primarily on empirical correlation. In this Note, the dynamics are considered in the absence of the breakup and the time scales of the dynamics are shown to be slightly different than those used in empirical correlation.

To model the acceleration and lateral deformation of a liquid drop when subjected to a relative air stream, we extend an approach used by Reinecke and Waldman.[5] Writing the drop's momentum equation lateral to the relative gas velocity, Reinecke and Waldman use a moment method to develop an equation for the lateral deformation of the drop. Assuming that the lateral liquid velocity is linear in the lateral coordinate, they obtain

$$\Delta d^2\Delta/dT^2 + 2(d\Delta/dT)^2 = 4C_p \tag{1}$$

where C_p is the pressure coefficient at the raindrop stagnation point, Δ is the ratio of the drop diameter D to the initial diameter D_0.

$$\Delta = D/D_0$$

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